Generalized transverse Bragg waveguides

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Abstract: A coupled-mode analysis of 2-D generalized transverse Bragg waveguides (GTBW) with tilted distributed Bragg reflectors is presented. As a result of the absence of inversion symmetry about a plane perpendicular to the guiding stripe, the modes supported by these guides are not separable into the familiar form of transverse standing wave and longitudinal traveling-wave components. This fundamental change in the modal description yields new and potentially useful guided-mode behavior. Expressions for the spatial distribution of the optical field, phase and group velocity, and the dispersion relation as well as applications of GTWB are presented.

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References and links


Familiar waveguides, including metallic, dielectric slab/fiber, transverse Bragg waveguides [1, 2] and photonic crystal (holey) fibers [3] are based on reflection with momentum transfer perpendicular to the guide direction at the transverse guide edges. In these cases, the guided mode can be functionally described as \( f(\vec{r}_\perp) \exp[i k_x x] \) where \( f(\vec{r}_\perp) \) is a function only of the...
transverse spatial coordinates, while \( k_\parallel \) is the wavenumber along \( x \), the guide direction. In 2D and 3D photonic crystal waveguides \([4,5]\) the functional form becomes \( f(\vec{r}) \exp(ik_\parallel \cdot \vec{r}) \) where \( f(\vec{r}) \) is now a function of all coordinates in accordance with Bloch’s theorem. The Bloch eigenmodes contain a rich diversity of potentially new waveguide behavior which has not yet been fully explored. The vector nature of the electromagnetic field coupled with the complexity of the possible structures typically pushes exploration of this new behavior to computationally intensive numerical simulations which inherently offer limited physical insight.

Recently, the coupled mode formalism \([6-8]\) has been applied to the problem of 2-D photonic crystal waveguides, offering analytical insight into the mechanisms at play in such waveguides. This analysis was restricted to the special case of a traditional Bragg waveguide with the Bragg planes parallel to the guiding direction, where \( f(\vec{r}) \) is a function of all the spatial coordinates, but the propagating plane wave remains a function of only the longitudinal coordinate, \( f(\vec{r}) \exp(ikx) \). The purpose of this report is to point out that this is only a special case of guiding and that the study of more general guided modes via coupled mode analysis, which cannot be described by the simple, separated functional form given above, is possible. Furthermore, these guided modes show interesting properties that are of both fundamental and technological interest.

One characteristic common to all of the above examples is reflection symmetry about sets of planes perpendicular to the guide. If this symmetry is broken, the separable functional dependence no longer holds, but there can still be guiding and energy transport along the guide. An important consequence of breaking this symmetry is that the mode is no longer symmetric to reflection in such a plane, thus reflection from any planar surface perpendicular to the guide, such as an exit facet, will not be guided.

Fig. 1. (a) Geometry of the tilted Bragg grating waveguide defining \( \theta_+ \), \( \theta_- \) and \( \theta_B \). The core of width \( W \) is between two Bragg grating regions of period \( \Lambda \) and width \( L \) (not shown) tilted by an angle \( \alpha \) from the guiding direction. (b) The wave vector diagram and (c) the sinusoidal variation of the index along the direction \( \zeta \) perpendicular to the variation of the Bragg grating.
The simplest example of such a structure is the tilted 2D Bragg waveguide shown in Fig. 1 where the planes of the Bragg structure are at an angle $\alpha$ to the guide direction ($x$). The Bragg reflectors extend for a distance $L$, not shown, both above and below the core. The variation of the dielectric constant along a line perpendicular to the grating (along $\zeta$ as shown in Fig. 1) is taken as $\varepsilon - \delta \sin(\zeta/\Lambda)$. In the core region, the field is composed of two unperturbed plane waves propagating with wave vectors $\tilde{k}_+$ and $\tilde{k}_-$. In the top Bragg reflector, these two waves are coupled such that the $\tilde{k}_+$ wave is reflected into the $\tilde{k}_-$ wave. At the bottom boundary these roles are reversed. As long as the magnitude of the product of the reflectivities is of order unity and the total round trip phase accumulation is an integer multiple of $2\pi$, a well-defined mode is obtained.

The momentum transfer on reflection from the tilted Bragg structure is not perpendicular to the guiding direction with the result that the standing wave associated with the reflection is also oriented at an angle to the guide and the simple, separable description is not valid. The requirements for a mode are that the power in the guide direction be invariant with x (assuming lossless materials and provided that the reflectivity from the claddings, controlled by the thickness of the cladding region, $L$, and the characteristics of the Bragg grating, is sufficiently large) and that it be localized in the vicinity of the core (the region of width $W$ between the Bragg reflectors in Fig. 1).

It is important to explicitly differentiate the tilted or generalized transverse Bragg waveguide (GTBW) from other varieties of Bragg waveguides. Figure 2 shows four different forms of Bragg waveguide Figs. 2(a)-2(d). Note the transverse Bragg waveguide in Fig. 2(a), also generically represents photonic crystal fibers, as well as VCSEL laser cavities (although the lasing direction in the VCSEL is normal to the Bragg planes, the structure is identical). The angled-grating DFB, or $\alpha$-DFB laser structure [9] in Fig. 2(b) also includes a similar grating confined waveguide with the Bragg planes parallel to the guiding direction. The angled grating in this case refers to the tilt of the grating with respect to the facets, not to the guide direction. The PC-DFB [10] in Fig. 2(c) is a 2-D extension of the $\alpha$-DFB, with the addition of distributed feedback in the waveguide direction. In all of these cases, the

Fig. 2. (a) Geometry of the transverse Bragg waveguide. (b) Geometry of the $\alpha$-DFB Laser. (c) Geometry of the PC-DFB Laser. (d) Geometry of the G TBW. (e) Family of Bragg planes used to analyze the 2-D PC waveguide in [6-8]. (f) Family of Bragg planes in the same 2-D PC waveguide which cannot be analyzed using the method outlined in [6-8].

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momentum transfer associated with the confinement is perpendicular to the waveguide optical axis. In Fig. 2(d), the structure of the GTBW is shown, with the arrows indicating the confinement occurring at an angle to the waveguide direction. Fig. 2(e) demonstrates the treatment of a 2-D photonic crystal waveguide in [6-8], considering the family of Bragg planes parallel to the waveguide axis using a separable \( f(\tilde{r})\exp(ik\tilde{x}) \) functional form. Figure 2(f) demonstrates a different set of Bragg planes in the same waveguide that cannot be analyzed via the approach in [6-8], but falls within the more general analysis presented here.

Assuming that the dielectric modulation is not too large, this situation is conveniently modeled with a simple coupled-mode theory [11-13]. For TE modes, the electric field vector is expressed as:

\[
E(x,z) = \hat{e}_y \left[ A_i(z - W/2, \theta_+) \exp \left[ i k_n \left( x \sin \theta_+ + \left( z - W/2 \right) \cos \theta_+ \right) \right] \right] + B_i(z - W/2, \theta_+) \exp \left[ i k_n \left( x \sin \theta_- - \left( z - W/2 \right) \cos \theta_- \right) \right]
\]

\[
E(x,z) = \hat{e}_y \left[ \exp \left[ i k_n \left( x \sin \theta_+ + \left( z - W/2 \right) \cos \theta_+ \right) \right] + R_i(\theta_+) \exp \left[ i k_n \left( x \sin \theta_- - \left( z - W/2 \right) \cos \theta_- \right) \right] \right]
\]

\[
E(x,z) = \hat{e}_y \left[ R_i(\theta_+) \exp \left[ i k_n \cos \theta_+ \left( B_i(z + W/2, \theta_-) \exp \left[ i k_n \left( x \sin \theta_+ + \left( z + W/2 \right) \cos \theta_+ \right) \right] \right] \right]
\]

\[
E(x,z) = \hat{e}_y \left[ \exp \left[ i k_n \left( x \sin \theta_- - \left( z + W/2 \right) \cos \theta_- \right) \right] \right]
\]

with \( k_n = \frac{2\pi}{\lambda} \) where \( n = \sqrt{\varepsilon} \), \( \lambda \) is the wavelength and \( \hat{e}_y \) is a unit vector in the \( y \)-direction. Coupling between the plane waves in the cladding is described by the spatially varying amplitude coefficients \( A_i(z, \theta_+) \) and \( B_i(z, \theta_-) \). Reflection at the core boundaries is described by the angle-dependent reflection coefficient \( R_i(\theta_+) \), expressed as the ratio of incident and reflected plane wave coefficients evaluated at the core boundary. Expressions for \( A_i(z, \theta_+), B_i(z, \theta_-), \) and \( R_i(\theta_+) \) are:

\[
A_i(z - W/2) = \frac{\left\{ -i \Delta \beta \sinh[H(z - W/2 - L)] \right\}}{i \Delta \beta \sinh(HL) + 2H \cosh(HL)} \exp \left[ \frac{i \Delta \beta}{2} (z - W/2) \right]
\]

\[
A_i(0) = 1
\]

\[
B_i(z - W/2) = \frac{2 \kappa^*}{\cos(\theta_-)} \frac{-\sinh[H(z - W/2 - L)]}{i \Delta \beta \sinh(HL) + 2H \cosh(HL)} \exp \left[ -i \frac{\Delta \beta}{2} (z - W/2) \right]
\]

\[
R_i = B_i(0) = \frac{2 \kappa^*}{\cos(\theta_-)} \frac{\sinh(HL)}{i \Delta \beta \sinh(HL) + 2H \cosh(HL)}
\]

with analogous expressions for \( A_p, B_p, \) and \( R_p \). Here,

\[
\Delta \beta = -\left[ k_n (\cos \theta_+ + \cos \theta_-) - K \cos \alpha \right], \quad \kappa = k_0 \delta
\]

\[
H = \frac{\left\{ \frac{k_n^2}{\cos \theta_+ \cos \theta_-} - \left( \frac{\Delta \beta}{2} \right)^2 \right\}}{\cos \theta_+ \cos \theta_-}
\]

\( \Delta \beta \) is the wave vector detuning from the Bragg resonance, \( \kappa \) is the coupling strength, and \( H \) is the inverse coupling length. Note that with the particular geometry we have adopted, \( \kappa \) is a real positive quantity and the reflection phase is zero for \( \Delta \beta = 0 \).
The self-consistency condition can be found by equating the field expressions in Eq. (1) at $z = -W/2$, yielding:

$$1 - R_r(\theta_+) R_b(\theta_-) \exp[i k_n (\cos \theta_+ + \cos \theta_-) W] = 0$$

(4)

For a given set of waveguide physical parameters, $n$, $W$, $\alpha$, and $\Lambda$, the roots of Eq. (4) yield the allowed bounce angles, $\theta_+$ and $\theta_-$ for self-consistent modes inside the waveguide core.

From Fig. 1 and the usual Bragg analysis,

$$\theta_+ = \theta_0 + \alpha + \delta \theta = \alpha' + \theta_0, \quad \theta_- = \alpha - \theta_0 + \delta \theta = \alpha' - \theta_0$$

(5)

and the self-consistency expression reduces to:

$$1 - R_r(\theta_0 + \alpha') R_b(\theta_0 - \alpha') \exp\left[i \frac{2 \pi W}{\Lambda} \left[\cos(\theta_0 + \alpha') + \cos(\theta_0 - \alpha')\right]\right] = 0$$

(6)

Here $\delta \theta$ is the detuning from the center of the Bragg resonance required to match the $2\pi$ modal phase condition (cf. Fig. 3(d)). The phase shift due to the round trip propagation across the guiding region, $2\pi W \cos(\alpha')/\Lambda$ is independent of wavelength since the wavelength variation of the Bragg angle, Eq. (5), just compensates the change in the optical path length. Thus, for any wavelength above the cutoff frequency, the relative position of the resonance within the bandpass is fixed, e.g. $\alpha' = \alpha + \delta \theta$ is independent of $\lambda$. It is straightforward to evaluate the phase and group velocities.

$$v_{\text{phase}} = \frac{\omega}{k} = \frac{c}{n \sin(\theta_b) \cos(\alpha')}$$

$$v_{\text{group}} = \frac{\partial \omega}{\partial k} = \frac{c \sin(\theta_b)}{n \cos(\alpha')}$$

(7)

At cutoff, corresponding to counterpropagating waves at the grating normal, $\lambda_{\text{cutoff}} \sim 2n\Lambda$, $v_p \rightarrow \infty$ and $v_g \rightarrow 0$; at the other extreme, as the wavelength is decreased, $\theta_b \rightarrow \pi/2 - \alpha'$ [and $\sin(\theta_b) \rightarrow \cos(\alpha')$] the group velocity reaches $c/n$, the velocity of light in the unbounded core medium. At wavelengths below this limit, one of the constituent plane waves is no longer confined to the guiding region and the mode is no longer bound. Thus, there is a short-$\lambda$ cutoff for $\alpha' \neq 0$ as well as the long-$\lambda$ cutoff associated with the Bragg condition.

Additional insight into the self-consistency relationship is provided by some simple numerical evaluations. We take $\lambda = 1300$ nm, $\varepsilon = (1.7)^2 = 2.89$ (appropriate to a high-index polymer), $\Lambda = 450$ nm, $\delta = 0.005$, $W = 2000$ nm and $L = 250$ $\mu$m and the grating tilt, $\alpha = 45^\circ$ - all readily accessible parameters. The top panel of Fig. 3(a) shows the magnitude of the reflectivity from the top Bragg reflector for these parameters with a stop band centered at $\theta_0 \sim 77^\circ$. The maximum field reflectivity is about $-0.5$ and varies across the stop band. The reflectivity at the second interface has just the inverse magnitude (e.g. $-0.2$) and the product, Fig. 3(b), the round trip reflectivity, is of order unity for incident angles spanning an angular bandwidth of $\pm\Delta$ about the geometrical Bragg angle. The round-trip reflectivity can be made arbitrarily close to unity throughout the entire bandwidth by increasing $xL$. Figure 3(c) shows the corresponding $2\pi$ phase variation across the stop band. Finally, Fig. 3(d) shows the magnitude of the self-consistency expression of Eq. 6 across this angular range. There is a
clearly defined root, and thus a mode with bounce angles $\theta_1=\theta_0+\alpha+\delta\theta = 76.16^\circ$ and $\theta_1 = 13.34^\circ$.

Referring to the $k$-space diagram in Fig. 4(f), the phase in the propagation direction progresses in the forward direction as the $\theta_+$ wave propagates from the bottom to the top interface, and then regresses in the opposite direction as the $\theta_-$ wave propagates from top to bottom. The resulting intensity pattern, Fig. 4, shows standing waves at an angle of $45^\circ$, corresponding to the tilt of the grating. The extent of the core region, without the Bragg gratings, is indicated by the horizontal dotted lines. For this modest coupling constant the mode extends well into the cladding regions; tighter confinement is available with higher $\delta$.

The two inner line plots are the variation of the intensity, $|E|^2$, along $x$ [Fig. 4(b)] and along $z$ [Fig. 4(c)]. The outer line plots [Figs. 4(d) and 4(e)] are the components of the Poynting vector in the respective directions. These figures confirm the central result. There is a mode confined to the core-guiding region that propagates along the $x$-direction without loss of power and with a wavelength-scale variation of the intensity along both the longitudinal and transverse directions.
Evaluation of the self-consistency equation as a function of wavelength yields the dispersion relation. The result of this process for a geometry for $\Lambda = 400$ nm and $W = 850$ nm, is shown in Fig. 5 in normalized frequency versus normalized longitudinal momentum with $\alpha$ as a parameter. The $\alpha = 0$ curve looks very much like a metal waveguide dispersion relation. In the absence of material dispersion, there is no upper frequency cutoff for the mode in a Bragg waveguide with $\alpha = 0$. As $\alpha$ varies, the position of the mode within the Bragg stop band varies in accordance with the self-consistency condition, such that the low frequency cut-off occurs at $\lambda_{\text{cutoff}} \approx 2n\Lambda$. For all waveguides with $\alpha > 0$, there is also a high frequency cutoff. In the limiting case 90° tilt, the upper and lower cutoff frequencies are degenerate and the structure exhibits a cavity resonance rather than a waveguide mode.

For applications, an operational definition of acceptable loss must be defined. This acceptable loss impacts both the minimum reflectivity magnitude (controlled by the cladding thickness and index contrast), and the angular bandwidth of the central high reflectance lobe (controlled by index contrast), and subsequently the range of possible values for the phase of reflectivity. In general, for cladding regions of length such that $HL>>1$, the round trip reflectivity is $\sim 1 - 4\exp\left[-kL/\sqrt{\cos \theta_+ \cos \theta_-}\right] \sim 1$, and the angular width of the high reflectivity pass band is given by

$$\Delta = 2\tan(\alpha)\left[\sqrt{1+kL \cos(\alpha)/[\pi \sin^2(\alpha) \cos(\theta_+) \cos(\theta_-)]} - 1\right]$$

with a nearly $2\pi$ swing in the allowed values of $\phi$. This fact has two consequences. First, waveguides with sufficiently
thick grating regions exhibit a continuum of allowable waveguide widths, just as in a traditional dielectric waveguide. Second, similar to symmetric dielectric slab waveguides, generalized transverse Bragg waveguides of vanishing width $W$ support guided modes.

In contrast, several recent publications [6-8] have suggested that Bragg guides have low loss modes only for specific $W = \Lambda/2, 3\Lambda/2, 5\Lambda/2, \ldots$. The difference can be traced to the assumption that $A\beta = 0$ so that the phase shift of the Bragg reflections is fixed at the center of the pass band and the only phase shift is given by the propagation term. Since the reflection phase shift can vary over a wide range for $HL \gg 1$, e.g. for $|R_r| \sim 1$, it is always possible to find the necessary phase shift for the existence of a mode, independent of $W$. In contrast to the assertions that there are only specific widths, $W$ that support modes, we find that there are some specific widths that do not support modes, but that these widths become vanishingly narrow as the length of the Bragg regions is increased. This is fully consistent with independent results that showed that bound modes exist in vertical-cavity surface-emitting structures for any active region width [14], as well as a numerical investigation revealing no waveguide width discretization in photonic crystal waveguides [5].

Guiding occurs in the $+x$ direction, when wave vectors $\vec{k}_+$ and $\vec{k}_-$ are coupled into each other via a momentum component of the angled grating as shown in Fig. 6(a) e.g when $\vec{k}_+ \pm \vec{K} = \vec{k}_-$. As a result of the rotational symmetry of the structure, guiding occurs in
the \(-x\) direction for \(-\vec{k}_+ \pm K = -\vec{k}_-\) as shown in Fig. 6(b). However, GTBW with \(\alpha \neq 0\), do not exhibit reflection symmetry about a plane perpendicular to the guide. After a reflection operation about the normal to the guide direction \(M(\vec{k}_+) = k_+ \rightarrow -k_+; k_y \rightarrow k_y; k_z \rightarrow k_z\), the wave vectors \(M(\vec{k}_+)\) and \(M(\vec{k}_-)\) are no longer coupled via a cladding momentum wave vector. Thus, the reflected waves are not a bound mode in the guide and the energy is lost into the cladding. The very important conclusion is that these guides are effectively unidirectional under reflection symmetry operations perpendicular to the guiding direction. Two potential applications are immediately suggested, optical amplifiers and high power fiber lasers. In both cases, back-scattered light limits the available performance either through increased amplified spontaneous emission (semiconductor amplifiers) or stimulated Brillouin scattering (fiber lasers).

As is the case for all Bragg guides, these guides lend themselves to single mode behavior for wide (large \(W\)) guides [15]. This is a result of the limited angular bandwidth of the Bragg reflector. A second mode is not allowed until both modes can be accommodated within the Bragg resonance, a substantially more demanding requirement than for index guides, where any angle larger than the critical angle is allowed. This permits larger area single-mode waveguides that are desirable for high power applications to ameliorate heating and laser damage limitations.

In conclusion, we have presented an analysis of tilted Bragg reflector waveguides that do not exhibit inversion symmetry about a plane perpendicular to the core. We demonstrate the possibility of low loss modes with non-separable traveling and standing wave components analyzed using a simple coupled-mode theory. These guides have potential application to situations where reflected waves are problematic – for example optical amplifiers where it is necessary to avoid spurious reflections and lasing. Finally, these modes are intimately connected to the modes of photonic crystal waveguides, and their further experimental and theoretical investigation will provide insight into this important new class of waveguides.

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